Time Allowed: **180 minutes**

## SECTION ONE - (3 points)

1. In how many ways can the number 1001 be written as the sum of two primes?

(A) none   (B) one   (C) two
(D) three   (E) more than three

2. Thor has seven stones and a hammer. Every time he hits a stone with the hammer it breaks into exactly five smaller stones. He does this several times. Which of the following numbers could be the number of stones he may end with?

(A) 17   (B) 20   (C) 21
(D) 23   (E) 25

3. If a, b are positive integers such that $2^a - 2^b = 992$, find the value of $a + b$.

(A) 15   (B) 16   (C) 17
(D) 31   (E) 32

4. How many squares with integer side length have the area of the form $\frac{n(3n+1)}{3}$, where n is a nonzero natural number?

(A) There is no such square   (B) 1 square   (C) 2 squares
(D) 3 squares   (E) An infinite number of squares

5. How many real solutions does the equation $2^x + 8^x = 2 \times 5^x$ have ?

(A) 0 solutions   (B) 1 solution   (C) 2 solutions
(D) 3 solutions   (E) an infinite number of solutions

6. The following two statements are true:
Some aliens are green, the others are purple.
Green aliens live only on Mars. Therefore, it logically follows that

(A) all aliens live on Mars   (B) only green aliens live on Mars.
(C) some purple aliens live on Venus.   (D) all purple aliens live on Venus.
(E) no green aliens live on Venus.

7. Four identical rhombuses and two squares are put together to make a regular octagon. What is the measure of the larger angle of each rhombus?
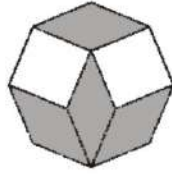
(A) 135° (B) 140° (C) 144°
(D) 145° (E) 150°

8. There are 65 balls in a box. 8 are white and the rest of the balls are black. In one move, at most 5 balls can be taken out of the box. It is not allowed to put any balls back into the box. What is the smallest number of moves needed to ensure that at least one white ball is taken out?

(A) 11 (B) 12 (C) 13
(D) 14 (E) 15

9. The faces of a rectangular brick have areas A, B and C as shown. What is the volume of the brick?
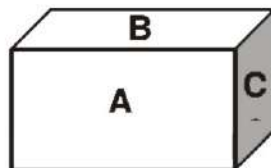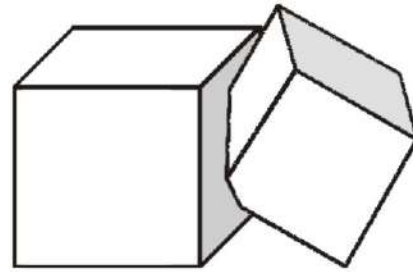

(A) ABC (B) $\sqrt{ABC}$ (C) $\sqrt{AB + BC + CA}$
(D) $\sqrt[3]{ABC}$ (E) $2(A + B + C)$

10. What is the sum of the 8 marked angles?
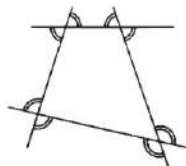
(A) 360° (B) 450° (C) 540°
(D) 720° (E) 900°
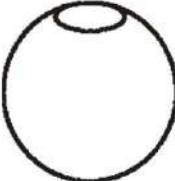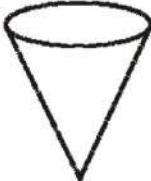
SECTION TWO - (4 points)

11. Two cubes of volumes V and W intersect. The part of the cube of volume V which is not common to the two cubes is 90% of its volume. The part of the cube of volume W which is not common to the two cubes is 85 % of its volume. What is the relationship between V and W?

(A) $V = \frac{2}{3} W$ (B) $V = \frac{3}{2} W$ (C) $V = \frac{85}{90} W$

(D) $V = \frac{90}{85} W$ (E) $V = W$

12. A vase is filled up to the top with water, at a constant rate. The graph shows the height h of the water as a function of time t.
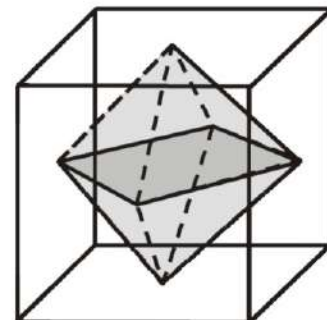Which of the following could be the shape of the vase?
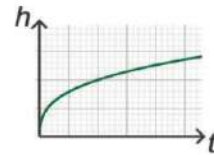
(A)   (B)   (C)

(D)   (E)

13. How many different pairs (a, b) of real numbers exist such that $a + b = ab = \frac{a}{b}$?

(A) 1   (B) 2   (C) 3
(D) infinitely many   (E) no one

14. An octahedron is inscribed in a cube of side length 1. The vertices of the octahedron are at the center of the faces of the cube. What is the volume of the octahedron?
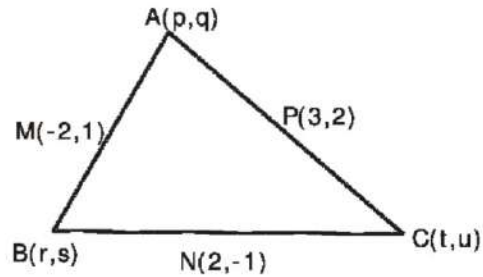
(A) $\frac{1}{3}$ (B) $\frac{1}{4}$

(C) $\frac{1}{5}$ (D) $\frac{1}{6}$

(E) $\frac{1}{8}$

15. The vertices of a triangle are $A(p, q)$, $B(r, s)$ and $C(t, u)$ as shown. The midpoints of the sides of the triangle are the points $M(-2, 1)$, $N(2, -1)$ and $P(3, 2)$. What is the value of $p + q + r + s + t + u$?

(A) 2   (B) $\frac{5}{2}$ (C) 3

(D) 5   (E) none of these

16. Five predictions were made before the football match between Real Madrid and Manchester United:

1) The game will not end in a draw;   2) Real Madrid will score;
3) Real Madrid will win;   4) Real Madrid will not lose;
5) Three goals will be scored.

What was the final score of the match Real Madrid - Manchester United if exactly three of the predictions came true?

(A) 3-0   (B) 2-1   (C) 0-3

(D) 1-2   (E) this situation is not possible

17. We cut out a regular pentagon from a lined piece of paper. In each step we rotate the pentagon counter clockwise around its centre by $21°$. The situation after the first step is shown. What will we see when the pentagon first fits back in the hole?

(A)   (B)   (C)

(D)   (E)
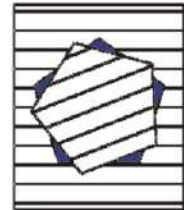
18. Which of these five numbers does not divide $18^{2017} + 18^{2018}$?

(A) 8   (B) 18   (C) 28
(D) 38   (E) 48

19. Three of the five cards shown are given to Nadia and the rest to Riny. Nadia multiplies the 3 values of her cards and Riny multiplies the 2 values of his cards. It turns out that the sum of the two resulting products is prime. What is the sum of the values of Nadia's cards?

(A) 12   (B) 13   (C) 15
(D) 17   (E) 18

20. Two rectangles are inclined to the vertical line at angles $40°$ and $30°$ as shown. What is the measure of the angle θ?

(A) $105°$ (B) $120°$
(C) $130°$ (D) $135°$
(E) None of these
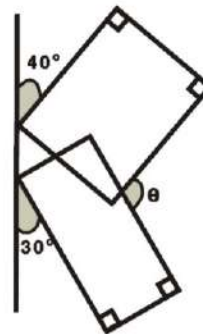
SECTION THREE - (5 points)

21. The prism in the picture is formed of two triangles and three squares. The six vertices are numbered from 1 to 6 in such a way that the sum of the four vertices of each square is the same for all three squares. Numbers 1 and 5 are already shown. What number is at the vertex labeled x?

(A) 2   (B) 3
(C) 4   (D) 6
(E) the situation is impossible
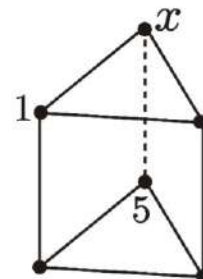
22. m and n are the roots of the equation $x^2 - x - 2018 = 0$. What is the value of $n^2 + m$?

(A) 2016   (B) 2017   (C) 2018
(D) 2019   (E) 2020

23. Four brothers named A, B, C and D have different heights. They state the following:

A: I am neither the tallest nor the shortest.
B: I am not the shortest.
C: I am the tallest.
D: I am the shortest.

Exactly one of them is lying. Who is the tallest?

(A) A   (B) B   (C) C
(D) D   (E) We do not have enough information

24. Let f be a function such that $f(x + y) = f(x)f(y)$ for all integers x and y.

If $f(1) = 1/2$, find the value of $f(0) + f(1) + f(2) + f(3)$.

(A) 1/8   (B) 3/2   (C) 5/2
(D) 15/8   (E) 6

25. A quadratic function $f(x) = x^2 + px + q$ is such that its graph intersects the x-axis and the y-axis in three different points. The circle through these three points intersects the graph of f in a fourth point. What are the coordinates of this fourth point?

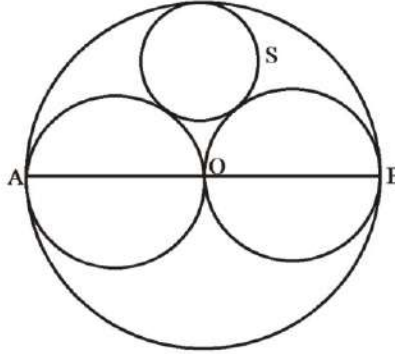(A) $(0, -q)$ (B) (p, q) (C) $(-p, q)$

(D) $(-\frac{q}{p}, \frac{q^2}{p^2})$ (E) $(1, p + q + 1)$

26. Let O be the center of segment $AB = 20$. Circle S is externally tangent to the circles with diameters AO and BO, and is internally tangent to a circle with diameter AB. Find the radius of circle S.

(A) $\frac{5}{2}$

(B) $\frac{10}{3}$

(C) 5

(D) $\sqrt{5}$

(E) $2\sqrt{5}$

27. How many real solutions does the equation $||4^x - 3| - 2| = 1$ have?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

28. $ABCDEF$ is a regular hexagon. G is the midpoint of AB. H and I are the points of intersection of the segments GD and GE with FC respectively. What is the ratio between the area of the triangle GIF and the area of the trapezoid $IHDE$?
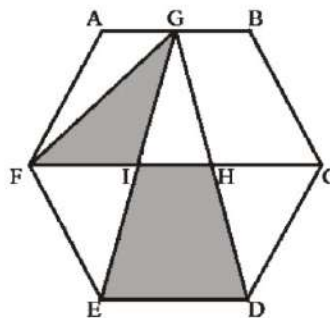
(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

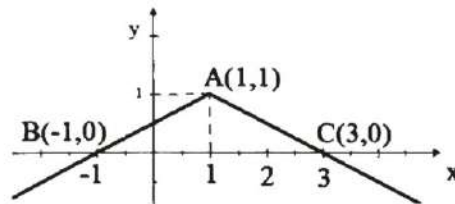(D) $\frac{\sqrt{3}}{3}$

(E) $\frac{\sqrt{3}}{4}$

29. There are 40% more girls than boys in a class. How many pupils are there in the class if the probability that a two-person delegation selected at random consists of a girl and a boy equals $\frac{1}{2}$?

(A) 20   (B) 24   (C) 36

(D) 38   (E) This situation is not possible.

30. The graph of the derivative function $f'(x)$ of a differentiable function $f: R \to R$ is depicted in the next figure. What is the value of $f(4) - f(0)$?

(A) 1   (B) 2   (C) 3

(D) 4   (E) 5

GOOD LUCK!